Equivalent properties for finite element analysis in composite design

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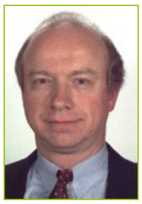

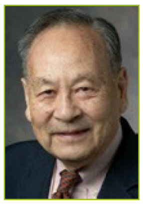

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The use of equivalent properties in composite design analysis can greatly shorten the overall CPU time and memory required for simulation. This article discusses the options and advantages of using equivalent properties for design.

Composite materials are defined traditionally in a ply-by-ply layup, with each layer definition consisting of a material type, an orientation angle (with respect to a reference direction) and a layer thickness. This ply layup can also be used to completely define a set of equivalent laminate properties that fully characterize the linear behaviour of the laminate due to in-plane loads and bending loads. The equivalent laminate properties define the relationship between stress and strain in terms of the membrane and bending contributions [2]. The submatrices A, D and B represent a partitioning of the stiffness matrix into membrane, bending and coupled membrane-bending effects.

$$[A] = \int_{-h/2}^{h/2} [Q]dz = \sum_{i=1}^{m} [Q']^{(i)}\left[z^{(i)} - z^{(i-1)}\right] = \text{in-plane matrix}$$

$$[B] = \int_{-h/2}^{h/2} [Q]zdz = \frac{1}{2}\sum_{i=1}^{m} [Q']^{(i)}\left[\left[z^{(i)}\right]^2 - \left[z^{(i-1)}\right]^2\right] = \text{coupling matrix}$$

$$[D] = \int_{-h/2}^{h/2} [Q]z^2dz = \frac{1}{3}\sum_{i=1}^{m} [Q']^{(i)}\left[\left[z^{(i)}\right]^3 - \left[z^{(i-1)}\right]^3\right] = \text{flexural matrix}$$

where $[Q']^{(i)}$ = off-axis stiffness of the i-th ply group with angle $\theta^{(i)}$

Fig. 2: Calculation of equivalent properties for laminated composites. Integrations are converted into a summation of contributions from each layer [2]

Combined equivalent properties vs. full ply-by-ply layup

Finite element analysis may use either the combined equivalent properties (classical laminate theory) or the full ply-by-ply layup to determine the laminate properties for the element using numerical integration. The modelling approach and mesh definition are the same for either procedure, but the nature of the results is rather different. The number of nodal degrees of freedom (DOFs) is exactly the same for either option.

However, the element formation and the stress recovery effort, both of which are functions of the number of layers in the laminate, are greatly simplified and the computational time reduced by the use of equivalent properties. With equivalent properties, the membrane strains and bending curvatures are available. These quantities can be used to compute the total strain at any position through the thickness. For the stress results, only equivalent membrane stress resultants and bending moment resultants are computed for each element. Stress results are not available at the layer level when using

	ABSOLUTE	NORMALIZED
STIFFNESS	$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \epsilon_j^o \\ k_j \end{Bmatrix}$	$\begin{Bmatrix} \sigma_i^o \\ \sigma_i^f \end{Bmatrix} = \begin{bmatrix} A_{ij}^* & B_{ij}^* \\ 3B_{ij}^* & D_{ij}^* \end{bmatrix} \begin{Bmatrix} \epsilon_j^o \\ \epsilon_j^f \end{Bmatrix}$
COMPLIANCE	$\begin{Bmatrix} \epsilon_i^o \\ k_i \end{Bmatrix} = \begin{bmatrix} \alpha_{ij} & \beta_{ij} \\ \tilde{\beta}_{ij} & \delta_{ij} \end{bmatrix} \begin{Bmatrix} N_j \\ M_j \end{Bmatrix}$	$\begin{Bmatrix} \epsilon_i^o \\ \epsilon_i^f \end{Bmatrix} = \begin{bmatrix} \alpha_{ij}^* & \frac{1}{3}\beta_{ij}^* \\ \tilde{\beta}_{ij}^* & \delta_{ij}^* \end{bmatrix} \begin{Bmatrix} \sigma_j^o \\ \sigma_j^f \end{Bmatrix}$

Fig. 1: Stiffness and compliance terms for laminate showing partitioning between membrane and bending effects [2]

equivalent properties. Conversely, when using the full layer-by-layer approach, a complete set of stress and strain results are calculated for each layer independently in the layer coordinate system. For this reason, the use of equivalent properties in design analysis can significantly shorten the overall CPU time and memory needed for the simulation. The options and advantages of equivalent properties for design are considered below. The homogenization process has benefits beyond the speedier calculations. Some of these benefits are described in a companion article by Tsai and Papila entitled: "Homogenization made easy with bi-angle thin-ply NCF" [1]. One of the key comparisons between the failure envelopes of ply-by-ply and equivalent models is shown below.

and degradation has not been validated. Element formation and stress recovery times both have a functional dependence on n, the number of layers in the laminate. However, the speedup will always be less than n because both element formation and stress recovery also have a constant time based on the element type. So both components have an operation count (and thus computation time) with a functional dependence of the form (a+bn). Even with this consideration, the speedup time is usually significant, especially when the number of layers in the laminate is large.

In addition to the element formation and stress recovery time, the other major contributor to the total solution time is the solution of the linear system of equations. The linear equation solution time is exactly

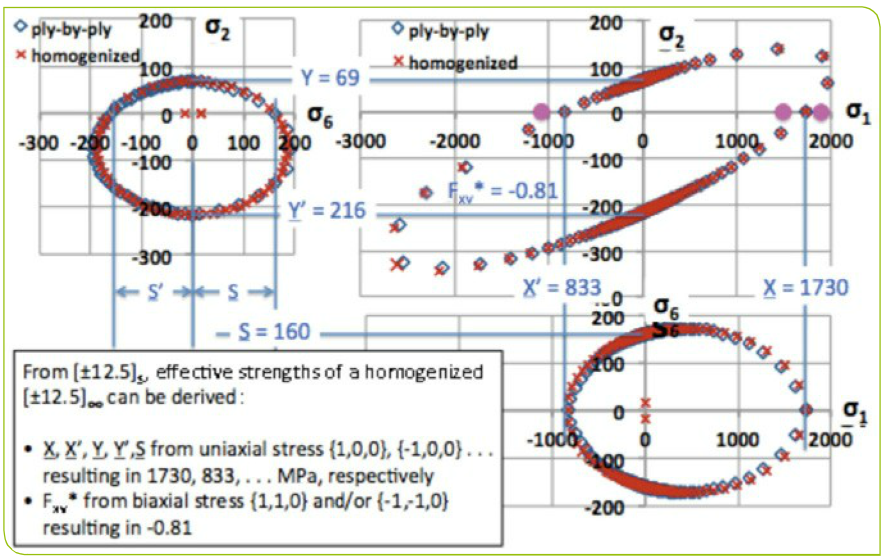

Fig. 3: Failure envelopes calculated using homogenized equivalent properties as compared with ply-by-ply properties [1]

Discussion

The use of equivalent properties is limited to linear analysis. This means that first ply failure (FPF) may be predicted when using equivalent properties, but nonlinear effects such as progressive damage or delamination are not considered at this time. Current approaches for deformation beyond FPF are empirical, i.e. a degradation or damage parameter is required to characterize the severity of the damage to the material. This sort of damage parameter is used in ply-by-ply modelling to calculate the incremental effects of damage and material degradation. An equivalent property approach for damage

the same for both options (equivalent properties and ply-by-ply properties), because the total number of degrees of freedom (DOF) is the same in both cases. Any overall speedup comes from the element formation and stress recovery speedup, and there is also a benefit from reduced memory requirements using equivalent properties because of the reduced solution data when using equivalent properties.

What does this mean for composite design? When designing composites, and in particular when it involves optimization, many solution iterations are often required. Each iteration involves a full solution of a particular composite problem. Significant speedups in total solution time and in memory requirements directly provide greater throughput in composite design.

For an n layer laminate, the use of effective laminate properties (classical laminate theory) vs. *n* layer ply-by-ply provides benefits by:

- Speedup of element formation time (faster x $(a_f+b_f n)/(a_f+b_f)$)
- Speedup of element stress recovery (faster x $(a_r+b_r n)/(a_r+b_r)$)

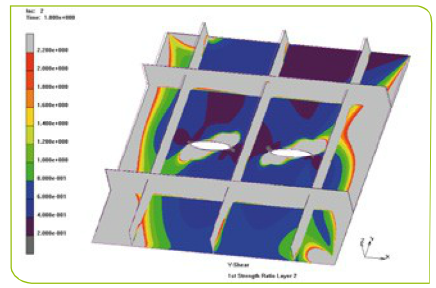

Fig. 4: Sample problem to compare finite element solution performance for equivalent properties vs. ply-by-ply properties. Shell model with 54543 shell elements, 55051 nodes and 64 layers

Fig. 4 shows an example involving a typical composite panel (reinforced shell with stringers and cutouts). The shell model has a total of 54543 shell elements, 55051 nodes and 64 layers. Comparing an analysis using equivalent properties with the layer-by-layer approach shows the advantages of using equivalent properties in this example:

- Element formation speedup: 11.4/1
- Stress recovery speedup: 17.1/1
- Equation solution speedup: 1/1
- Total solution time speedup: 5.0/1
- Total memory reduction: 3.8/1

Each iteration of a design optimization analysis will show similar improvement. Overall solution and memory reduction leads to an approximate speedup of 5 times. This level of benefit shows the power of using equivalent properties for design. ■

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References

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[2] Tsai, S. W. (2008). Theory of Composites Design. Think Composites.