Fracture Mechanics
Overview & Basics

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Motivation

• Brittle failure at low stress in low-strength steels
  – WWII era Liberty Ships
    • Welded structures
    • Ductile/Brittle transition temperature too high

• The occurrence of failure at low stress in high-strength materials spurred Fracture Mechanics further

• NBS economic study in 1978 estimated the cost of fracture at $119 Billion in the U.S. (about 4% of the GNP)
Fracture v Strength Criteria

Figure 1.1. Major Design Criteria [1]
Historical Development

• A.A. Griffith work on brittle fracture
• G.R. Irwin extended Griffith’s work to ductile materials
Work of Griffith

• A. A. Griffith started his work in around the 1920s. At this time, it was accepted that the theoretical strength of a material was taken to be $E/10$, where $E$ is Young's Modulus for the particular material. He was only considering elastic, brittle materials, in which no plastic deformation took place. However, it was observed that the true values of critical strength was as much as 1000 times less than this predicted value, and Griffith wished to investigate this discrepancy.

• He discovered that there were many microscopic cracks in every material which were present at all times. He hypothesized that these small cracks actually lowered the overall strength of the material because as a load is applied to these cracks, stress concentration is experienced.
Work of Irwin

- G. R. Irwin, in the 1950s, began to see how the theory outlined by Griffith would apply to ductile materials.
- He determined that there was also a certain energy from plastic deformation that had to be added to the strain energy originally considered by Griffith in order for the theory to work for ductile materials.
- Irwin developed the concept of the strain energy release rate.
Concepts & Principles

• Crack tip stress distribution
  – $1/\sqrt{r}$ singularity at the crack tip (LEFM)

• Energy Release Rate is related to the Stress Intensity Factor

• Condition at the crack tip is characterized by the Stress Intensity Factor, $K_I$

• Fracture toughness is a material property, $K_{IC}$. 
Three Modes of Cracking

- Mode I - opening mode
- Mode II - in-plane shearing/sliding mode
- Mode III - out-of-plane shearing/tearing mode

Deal with Mode I most frequently
Stress Concentration

**Maximum Stress**

\[
\sigma_M = 2\sigma_0 \sqrt{\frac{a}{\rho_t}} + \sigma_0
\]

**Stress Concentration Factor**

\[
K = \frac{\sigma_m}{\sigma_0} = 2 \sqrt{\frac{a}{\rho_t}} + 1.0
\]
Crack Tip Stresses

\[ K_I = C \sigma \sqrt{\pi a} \quad \text{[stress}\ast\text{length}^{0.5}] ; \]

Ex: (ksi-\(\sqrt{\text{in}}\), MPa-\(\sqrt{\text{m}}\))

- Near-tip stress field (LEFM)

\[
\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]

\[
\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]

\[
\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \left( \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) \]

**FIGURE 8.5** Distribution of stresses in vicinity of crack tip.
Some Applications of Fracture Mechanics

• Remaining life as a function of crack size
• Crack size that can be tolerated (critical crack size)
• Time for a crack to grow from an initial size to the critical size (inspection interval)
Test Techniques for Fracture Properties

• Compact Tension (CT) specimen

• Charpy impact specimen
### Stress Intensity Factors for CT

**Type**

1. Compact specimen
   - **C(T)**
   - **Configuration**
   - \( K = \frac{P}{BW^{\frac{1}{2}}} f(a/W) \)
     - where \( f(a/W) = \frac{(2 + a/W)}{(1 - a/W)^{\frac{3}{2}}} \left[ 0.886 + 4.64a/W - 13.32(a/W)^2 + 14.72(a/W)^3 - 5.6(a/W)^4 \right] \)

2. Disk-shaped compact specimen
   - **DC(T)**
   - **Configuration**
   - \( K = \frac{P}{BW^{\frac{1}{2}}} f(a/W) \)
     - where \( f(a/W) = \frac{(2 + a/W)}{(1 - a/W)^{\frac{3}{2}}} \left[ 0.76 + 4.8a/W - 11.58(a/W)^2 + 11.43(a/W)^3 - 4.08(a/W)^4 \right] \)

3. Wedge opening loaded specimen
   - **WOL**
   - **Configuration**
   - \( K = \frac{P}{BW^{\frac{1}{2}}} f(a/W) \)
     - where \( f(a/W) = \frac{(2 + a/W)}{(1 - a/W)^{\frac{3}{2}}} \left[ 0.8072 + 8.858(a/W) - 30.23(a/W)^2 + 41.008(a/W)^3 - 24.15(a/W)^4 + 4.951(a/W)^5 \right] \)
Miniature Specimens

• Trend: Development of techniques using miniature specimens

• Example: Small Punch procedure for generating constitutive properties & fracture toughness
Piping Design: Leak Before Break Criteria

- Desirable for pipe to be able to grow a crack through-wall (leak) without unstable crack growth (break).
- Leak provides an opportunity to detect the crack before it becomes long enough to be critical.
Pipeline-Unstable Crack Growth

- 16-inch underground natural gas line
- 300 psi internal pressure
- Poor quality welds (ERW pipe)
- Fast fracture of a 40-ft. section after initial weld defects grew through fatigue to critical size
- Resulting fire & explosions demolished the plant
Fatigue Growth

- **Cyclic stress intensity factor**
  \[
  \Delta K = K_{\text{max}} - K_{\text{min}} = Y\Delta \sigma \sqrt{\pi a}
  \]

- **Paris law**
  \[
  \frac{da}{dN} = A(\Delta K)^m
  \]

- **Integrate to get Cycles to failure**
  \[
  N_f = \int_0^N dN = \int_{a_0}^{a_f} \frac{da}{A(Y\Delta \sigma \sqrt{\pi a})^m} = \frac{1}{A\pi} \frac{m/2}{(\Delta \sigma)^m} \int_{a_0}^{a_f} \frac{da}{Y^m a^{m/2}}
  \]
Fatigue Fracture Surface

- Final tensile failure
- Stage II
- Stage I
- Direction of alternating stress

- Origin of fatigue crack
- Fatigue zone
- Rupture zone
- Concave marks known as clamshells or step marks
- Herringbone pattern or granular trace

- Origin of fracture
- Granular surface
- Rubbed surface
- Clamshell marking
Conclusions

- Cracks are everywhere, it is just a matter of how large!
- Fracture design criteria are required more often in modern design
- More efficient designs with lower factors of safety drive this trend
Selected References: Fracture Mechanics


